Appendix for DataColada[39].
Derivation that if test-retest correlation for a dependent variable is r<.5,
subtracting baseline lowers power.

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Let's consider a two-cell design, treatment vs control, with dependent variable: *y*
Let y_2^t and y_2^c be the means for treatment and control respectively in the *after* period.
Let y_1^t and y_1^c be the means for treatment and control respectively in the *before* period.

The between subject difference
(1) $B = y_2^t - y_2^c$

The mixed-design test subtracts the baseline
(2) $M = y_2^t - y_2^c - \underbrace{(y_1^t - y_1^c)}_{baseline}$

The expected difference is the same, E(B)=E(M), because with random assignment we have $E(y_1^t - y_1^c)=0$

This makes sense, we don't expect differences at baseline, so we expect the same with B or M

How about the standard error of B and M?

Let's make things easy. Assume all variances are the same:
(3) $VAR(y_2^t)=VAR(y_1^t)=VAR(y_2^c)=VAR(y_1^c)=V$
(4) $COV(y_2^t, y_1^t)=COV(y_2^c, y_1^c)=C$

(note: because of random assignment $COV(y_2^c, y_2^t)= COV(y_1^c, y_1^t)=0$)

Recall the high-school formula for variance of sum of random variables:
(5) VAR(a-b)=VAR(a)+VAR(b)-2COV(a,b)

We want to compute the variance of the B (between) and M (mixed design) estimates:

$VAR(B)=VAR[(y_2^t - y_2^c)]]$
=2V

VAR(M)=VAR[(Y2-Y1) – (X2-X1)]
4V-4C

Mixed and Between subject design have the same sample size and the same effect size, hence Mixed has more power iff its variance is smaller than Between's.

For VAR(B)>VAR(M) we need
2V>4V-4C
Which occurs if
4C>2V
Which occurs if
C/V>1/2

C/V, the covariance over the variance, is the correlation, so:

The Mixed design has a smaller variance and hence greater power iff r>.5